BEHAVIOR OF A PLANAR ENCLOSED JET OF VISCOPLASTIC LIQUID

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The behavior of an enclosed jet of viscoplastic liquid is considered by reference to the boundary-layer equations in terms of Mises variables.

The following is the form for the equations for a steady-state planar boundary layer on a Herschel—Buckley medium at constant pressure in an external flow [1]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left[\tau_0+k\left(\frac{\partial u}{\partial y}\right)^n\right],\ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

The boundary conditions are

$$\frac{\partial u}{\partial y}=0 \text{ at } \quad y=0,\ u\to 0 \text{ for } y\to\pm\infty. \tag{2}$$

The integral condition is

$$\int_{-\infty}^{+\infty}\rho u^2 dy=K_0=\mathrm{const}. \tag{3}$$

We transfer to Mises variables [2] in (1),

$$\int_0^x \omega(x)\,dx=\xi,\qquad \int_0^y u(x,\ y)\,dy=\eta, \tag{4}$$

which gives

$$\frac{\partial z}{\partial\xi}=\frac{k}{\rho}\frac{u}{\omega}\frac{\partial}{\partial\eta}\left[\frac{\partial z}{\partial\eta}\right]^n, \tag{5}$$

where $z = u^2/2$; in (5) we put $\omega = \sqrt{2}(kn/\rho)(C/\xi)^{(3n-2)/2}$, where C is a constant to be determined. The solution to (5) is sought in the following form from the analogy with an enclosed jet of Newtonian liquid [2]:

$$z=\frac{C}{\xi}\varphi(\zeta),\quad \zeta=\eta\xi^{-1/2}. \tag{6}$$

Here the constant C is defined by the integral condition (3): $C=\left[K_0/\sqrt{2}\rho\int_{-\alpha}^{+\alpha}\sqrt{\varphi}\,d\zeta\right]^2$, where $\pm\alpha$ are the roots of $\varphi(\zeta)=0$. We substitute the values of z and ζ from (6) into (5) to get

$$(\varphi')^{n-1}\varphi''+\frac{\zeta}{2\sqrt{\varphi}}\varphi'+\sqrt{\varphi}=0. \tag{7}$$

The prime and two primes denote respectively the first and second derivatives with respect to ζ. The boundary conditions for φ are given by (2) as follows together with the conditions of symmetry and smoothness of the distribution of the longitudinal velocities in the jet [2]:

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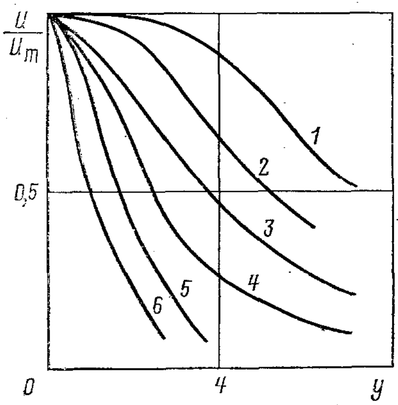

Fig. 1. Distribution of the longitudinal velocity u/u_m over the width y of a jet of viscoplastic liquid: 1) n = 0.2; 2) 0.3; 3) 0.4; 4) 0.6; 5) 1.0; 6) 3.0.

$$\varphi = 1 \text{ and } \varphi' = 0 \text{ at } \zeta = 0. \tag{8}$$

Equation (7) with the boundary conditions of (8) has the solution

$$\varphi = \left[1 + \frac{1 - \frac{1}{2n}}{1 + \frac{1}{2n}} \zeta(-n\zeta)^{\frac{1}{n}}\right]^{\frac{1}{1 - \frac{1}{2n}}}. \tag{9}$$

The longitudinal velocity component in Mises variables is

$$u = \sqrt{2z} = \sqrt{2C}\,\xi^{-1/2}\varphi^{1/2}. \tag{10}$$

To derive the solution to (1) it is necessary to transfer back to the physical coordinates x and y in accordance with (4):

$$y = \int\limits_0^\eta \frac{d\eta}{u}, \quad \sqrt{2}\,\frac{kn}{\rho} \int \left(\frac{C}{\xi}\right)^{(3n-2)/2} dx = \xi, \tag{11}$$

and the latter equation gives an explicit relation between x and ξ:

$$\xi = \left\{\sqrt{2}\,\frac{kn}{\rho}\left(\frac{2n-1}{2} + 1\right) C^{(2n-1)/2}\right\}^{2/3n} x^{2/3n}. \tag{12}$$

This solution agrees with Loitsyanskii's solution [2] for a Newtonian liquid for n = 1. The longitudinal velocity at the axis of the jet ($\zeta = 0$) is shown by (10) and (12) to fall in accordance with $u_m \sim x^{-1/3n}$, i.e., much more rapidly than for a Newtonian liquid (n = 1), and in the same way as for a nonlinearly viscous liquid ($u_m \sim x^{-1/3n}$) [1]. The shape of the jet is also dependent on the rheological factor, as is evident from substituting the u of (10) into (3) on the basis of (12):

$$b(x) = \text{const}\, x^{2/3n}. \tag{13}$$

The boundary of the jet has a convex outer form in the case $n > 2/3$, as is evident from the behavior of db/dx as x increases. For n = 2/3 the boundaries are rectilinear, and for $n < 2/3$ they take the form of diverging parabolas. Therefore, the ejection capacity of the jet increases as the rheological parameter n decreases, which corresponds to an increase in the pseudoplastic behavior. This is also confirmed by the velocity distribution over the width of the jet calculated from (10) on the basis of (9) and (11) and shown in Fig. 1. For small values of n, which correspond to a substantial jet width, it becomes incorrect to consider the problem within the framework of boundary-layer theory.

In calculating the width one takes the two real symmetrical roots $\pm\alpha$ of $\varphi(\zeta) = 0$; a nonlinear liquid also shows a tendency for the ejection capacity to increase as n decreases, along with the change in the geometry [1].

NOTATION

x, y, longitudinal and transverse coordinates; u, v, longitudinal and transverse velocities; ρ, density; τ, shear stress; n, rheological parameter characterizing the non-Newtonian behavior; k, consistency measure; K_0, momentum; ξ, η, Mises variables [formula (4)]; b(x), jet boundary; u_m, maximum velocity in the section.

LITERATURE CITED

1. Z. P. Shul'man, Convective Heat and Mass Transfer for a Rheologically Complex Liquid [in Russian], Énergiya, Moscow (1975), pp. 120-127.
2. L. G. Loitsyanskii, Laminar Boundary Layers [in Russian], Gosizdat. Fiz.-Mat. Literatury, Moscow (1962), pp. 23-26, 39-42.

RESISTANCE OF A BODY OF ROTATION WITH A CENTRAL HOLE IN A SUPERSONIC FLOW

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Simulation is applied to a body with conical inlet and outlet to determine the dependence of the resistance on the geometrical characteristics and the Mach number of the supersonic flow.

A body of rotation with central flow belongs to the class for which the aerodynamic features in supersonic flow are largely determined by the variations in the shock-wave structure as the flow speed varies. We have examined the resistance law for such a body for Mach numbers in the unperturbed flow in the range $1.2 \le M_\infty \le 6$ (here and subsequently, subscript ∞ denotes a parameter in the unperturbed flow). The study is numerical by means of Godunov's nonstationary difference scheme [1], which is used with an algorithm for constructing oblique-angle cells and a system of Euler equations written in a cylindrical coordinate system. The standard boundary conditions are used for the incident unperturbed flow, at the symmetry axis, and at the solid surfaces; at the other open surfaces we use the conditions for zero values of the derivatives of the gasdynamic parameters along the normals to these surfaces. The distance from the surface of the body to the boundaries of the working region was selected during the numerical experiments, along with the nonuniformity in the distribution of the nodal lines in this region.

The body (Fig. 1a) is a cylinder of diameter D and of length L = 1.5 or 2.25D with a hole of diameter $d \le 0.9D$ with conical inlet and outlet. The cone angles θ_1 at the inlet were 2, 20, and 90°, while those at the outlet were θ_2 = 20, 26, and 90° (the form $\theta_1 = \theta_2 = 90°$ is a cylinder with a central hole and no sharp edges at the inlet and outlet). For $\theta_1 \ne 90°$ we consider the forms with sharp and blunt edges at the inlet: $d_1 = D$; $d_1 = 0.95D$, where d_1 is the diameter at the leading end section of the cylinder, which determines the degree of blunting of the edges. The edge blunting was taken as zero at the exit from the channel for $\theta_2 \ne 90°$.

Parts b and c of Fig. 1 show the shockwave structures (lines of constant pressure P/P_∞) as found near a body whose geometry was represented by the following set of parameters: d = 0.8D; d_1 = D; L = 1.5D; $\theta_1 = \theta_2 = 20°$ for the case $M_\infty = 2$ (b) and $M_\infty = 6$ (c). It is evident

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